Stress-Energy Tensors for Higher Dimensional Gravity

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Abstract

We calculate, in the context of higher dimensional gravity, the stress-energy tensor and Weyl anomaly associated with anti-de Sitter and anti-de Sitter black hole solutions. The boundary counter-term method is used to regularize the action and the resulting stress-energy tensor yields both the correct black hole energies as well as a vacuum energy contribution which is interpreted as a Casimir energy. This calculation is done up to $d = 8$ (d being the boundary dimension). We confirm some results for $d < 8$ as well as comment on some new results. All results for $d = 8$ are new.

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1 Introduction

There has been much debate in general relativity as to how to assign the stress-energy contribution due to the gravitational field. Early works in

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this field include Einstein's introduction of the pseudo- tensor [1] however, this lacks invariance which should be present in a covariant theory such as relativity. Levi-Civita's argument [2] that the stress-energy tensor, as defined by the Einstein tensor, plays the role of "balancing out" space-time's stress energy is a more natural interpretation. More recent work on the subject may be found in [3].

The motivation for the counterterm subtraction method as found in [4], [5] and [6] is not so much to define a stress-energy tensor for gravity but to regularize the gravitational action for spacetimes with constant energy densities due to a cosmological term:

$$S = \frac{1}{16\pi}\int_B d^Dx\,\sqrt{|g|}\left(R+\alpha^2\left(d(d-1)\right)\right) - \frac{1}{8\pi}\int_{\partial B} d^dx\sqrt{|\gamma|}K + \frac{1}{8\pi}S_{(ct)}(\gamma). \tag{1}$$

[1] Here, d is the dimension of the boundary (∂B), of the D-dimensional bulk spacetime (B). γ is the determinant of the boundary metric, γ_{ab} and K is the trace of the extrinsic curvature, K_{ab} of the boundary. The first term is the usual Einstein Hilbert term, the second the Gibbons-Hawking surface term and the third is the counterterm action which removes the stress-energy tensor divergences which result from the previous terms.

Varying the first two terms with respect to the boundary metric yields the unrenormalized stress-energy tensor [8]:

$$T^{ab}_{(unren)} = \frac{1}{8\pi}\left(K^{ab} - K\gamma^{ab}\right). \tag{2}$$

The final term in (1) may be constructed in two ways. There is the background subtraction method of Brown and York [8] where one chooses for $S_{(ct)}$ the action of a spacetime with the same intrinsic geometry as the spacetime of interest. For black holes of mass M for example, a natural choice would be the $M=0$ limit of the original spacetime (for example see [9]). Another method, which we will use here, involves constructing $S_{(ct)}$ from curvature invariants of γ^{ab} [4] and therefore bulk equations of motion will not be affected. This method allows definitions of conserved quantities without the introduction of a spacetime which is external to the one under study. Also, this method is useful when considering spacetimes which do not

[1] Conventions follow that of [7] and α is $1/l$ where l is the radius of the anti-de Sitter spacetime.

have a natural reference background to which a comparison may be made or when non-trivial topologies are present.

1.1 The boundary counterterm method.

In this section we will briefly review the method of sucessive boundary counterterms which was first introduced in [4] and [6]. Schematically, the counterterms may be written as an expansion in inverse powers of α:

$$S_{(ct)} = \alpha \left(S^{(0)} + \alpha^{-2} S^{(1)} + ... \right). \tag{3}$$

The resulting total stress-energy tensor is then just

$$T^{ab}_{(finite)} = T^{ab}_{(unren)} + T^{ab}_{(ct)}, \tag{4}$$

where $T^{ab}_{(ct)}$ comes from the variation of $S_{(ct)}$ with respect to the boundary metric γ_{ab}.

The appropriate counterterms are uniquely determined by demanding that the resulting stress-energy tensor be finite. This finite tensor must then reproduce the correct conserved quantities for known solutions.

If we write the line element of the boundary in ADM form where the hypersurfaces (Σ) are spacelike surfaces of constant t:

$$ds^2_{\partial B} = -N^2_\Sigma dt^2 + \sigma_{ab}(dx^a + N^a_\Sigma dt)(dx^b + N^b_\Sigma dt), \tag{5}$$

then the energy of the spacetime is obtained from the energy density as

$$M = \int_\Sigma d^p x \sqrt{\sigma} \sqrt{|g_{tt}|} u^a u^b T_{ab(finite)}. \tag{6}$$

Where u^a is the unit normal to Σ.

Although we will be primarily interested in energies, other conserved quantities may be similarly calculated by exploiting other Killing symmetries. The integral in (6) will diverge due to the asymptotic behaviour of the metric determinant unless $T_{ab(finite)}$ tends to zero for large r in such a way as to remove divergences. It is this requirement for the conserved quantities to be finite which uniquely determines the form of the counterterms.

2 Calculations

Setting $S_{(ct)} = \int_{\partial B} \sqrt{|\gamma|}\mathcal{L}_{(ct)}$ the following Lagrangian is required to remove divergences up to d=8 [5]:

$$\mathcal{L}_{(ct)} = -\alpha(d-1) - \frac{R}{2\alpha(d-2)} - \frac{1}{\alpha^3 2(d-2)^2(d-4)}\left(R^{ab}R_{ab} - \frac{d}{4(d-1)}R^2\right)$$
$$+\frac{1}{\alpha^5(d-2)^3(d-4)(d-6)}\left[\frac{3d+2}{4(d-1)}RR^{ab}R_{ab} - \frac{d(d+2)}{16(d-1)^2}R^3 - 2R^{ab}R^{cd}R_{acbd}\right.$$
$$\left. + \frac{d-2}{2(d-1)}R^{ab}R_{;b;a} - R^{ab}R_{ab;c}{}^{;c} + \frac{1}{2(d-1)}R\,R_{;c}{}^{;c}\right]. \quad (7)$$

The action is to be varied with respect to the boundary metric, $\frac{\delta S_{(ct)}}{\delta \gamma_{ab}}$, and this yields the following for the stress-energy counterterm:

$$
\begin{aligned}
T^{ab}_{(ct)} &= -\alpha(d-1)\gamma^{ab} + \frac{1}{\alpha(d-2)}G^{ab} + \frac{1}{\alpha^3(d-2)^2(d-4)}\left[\frac{1}{2}\left(\frac{d}{4(d-1)}R^2 - R^{cd}R_{cd}\right)\gamma^{ab}\right. \\
&- \left.\frac{d}{2(d-1)}RR^{ab} + 2R_{cd}R^{cadb} - \frac{d-2}{2(d-1)}R^{;a;b} + R^{ab}{}_{;c}{}^{;c} - \frac{1}{2(d-1)}R_{;c}^{;c}\gamma^{ab}\right] \\
&+ \frac{2}{\alpha^5(d-2)^3(d-4)(d-6)}\left\{\frac{3d+2}{4(d-1)}\left[-G^{ab}R^{cd}R_{cd}\right.\right. \\
&- 2RR^{ca}R^b_c + (R^{cd}R_{cd})^{;a;b} - \gamma^{ab}(R^{cd}R_{cd})_{;e}^{;e} + 2(RR^{bc})^{;a}_{;c} - \gamma^{ab}(RR^{cd})_{;c;d} \\
&- \left.(RR^{ab})^{;c}_{;c}\right] - \frac{d(d+2)}{16(d-1)^2}\left[\frac{1}{2}R^3\gamma^{ab} - 3(R^2R^{ab} + (R^2)^{;a;b} - (R^2)^{;c}_{;c}\gamma^{ab})\right] \\
&- 2\left[\frac{1}{2}R^{ef}R^{cd}R_{ecfd}\gamma^{ab} - 3R^{ae}R^{cd}R^b_{ced} + (R^{ac}R^{bd})_{;c;d} - (R^{ab}R^{cd})_{;c;d}\right. \\
&+ \left.2(R^b_{cfd}R^{cd})^{;a;f} - (R_{fcgd}R^{cd})^{;g;f}\gamma^{ab} - (R_{cd}R^{cadb})^{;e}_{;e}\right] \\
&+ \frac{d-2}{2(d-1)}\left[\frac{1}{2}R^{cd}R_{;d;c}\gamma^{ab} - 2R^{;(ca)}R^b_c + \frac{1}{2}\left[2R^{(;c;b);a}{}_{;c} + -R^{;c;d}{}_{;c;d}\gamma^{ab}\right.\right. \\
&- \left.\left.(R^{;b;a})^{;c}_{;c}\right] + (R^{;a}R^{bc})_{;c} - \frac{1}{2}(R^{ab}R^{;c})_{;c} + (R^{cd}{}_{;c;d})^{;a;b}\right] \\
&- \left[\frac{1}{2}R^{cd}(R_{cd})^{;e}_{;e}\gamma^{ab} + 2(R^{bc\,e}{}_{;e})^{;a}_{;c} - (R^{ab}{}_{;c}{}^{;c})^{;e}_{;e} - 2R^{a}_{;c}{}^{;c}R^{cb} - R^{cd}R_{cd}^{;a;b}\right. \\
&- \frac{1}{2}(R^{cd}{}_{;e}{}^{;e})_{;c;d}\gamma^{ab} - (R^{ac;f}R^b_c)_{;f} - 2(R^{fc;a}R^b_c)_{;f} + 2(R^{bc;a}R^d_c)_{;d} + (R_{cd}^{;a}R^{cd})^{;b} \\
&- \left.(R_{cd}^{;e}R^{cd})_{;e}\gamma^{ab} - 2(R^a_c)^{;e}_{;e}R^{cb} + (R^a_cR^{bc;e})_{;e}\right] + \frac{1}{2(d-1)}\left[-G^{ab}R^{;c}_{;c} - R\,R^{;b;a}\right. \\
&+ 2(R^{;c}_{;c})^{;a;b} - 2(R^{;c}_{;c})^{;e}_{;e}\gamma^{ab} + (R\,R^{;a})^{;b} - \frac{1}{2}(R\,R^{;c})_{;c}\gamma^{ab} + R^{;c}_{;c}R^{ab} \\
&+ \left.\left.(R^{cd}{}_{;c;d})^{;e}_{;e}\gamma^{ab}\right]\right\}
\end{aligned}
\tag{8}
$$

with G^{ab} being the Einstein tensor formed from the boundary metric γ^{ab}. This, albeit rather large, expression will allow us to compute the conserved charges of the spacetime [2]. We choose to study the higher dimensional Schwarzschild-anti-deSitter black holes whose geometry in Schwarzschild co-

[2]For calculations of conserved charges and Casimir energies of $d = 4$ Kerr-AdS spacetimes see [12]

ordinates is given by:

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{p-1} + \alpha^2 r^2\right) dt^2 + \frac{dr^2}{\left(1 - \left(\frac{r_0}{r}\right)^{p-1} + \alpha^2 r^2\right)} + r^2 d\Omega_p^2 \quad (9)$$

where $p = d - 1$. $d\Omega^2$ is the metric on unit p-spheres which, for arbitrary p, is given by:

$$d\Omega_p^2 = \left[d\theta_0^2 + \sum_{n=1}^{p-1} d\theta_n^2 \left(\prod_{m=1}^{n} \sin^2 \theta_{m-1} \right) \right]. \quad (10)$$

Using (6) we calculate the masses of 5, 7 and 9 dimensional black holes which may be summarized as follows:

$$\begin{aligned} M_5 &= \frac{3\pi r_0^2}{8} + \frac{3\pi}{32\alpha^2} \\ M_7 &= \frac{5\pi^2 r_0^4}{16} - \frac{5\pi^2}{128\alpha^4} \\ M_9 &= \frac{7\pi^3 r_0^6}{48} + \frac{35\pi^3}{3072\alpha^6}. \end{aligned} \quad (11)$$

In the limit of vanishing black hole mass, $r_0 = 0$, we have a pure vacuum state which has non-zero energy. It was noted in [4] and [10] that, in light of the AdS/CFT correspondence [11], the second term in M_5 may be interpreted as the Casimir energy of the dual field theory which is $d = 4$, $\mathcal{N} = 4$, SUSY Yang Mills theory. It is interesting to note that in seven dimensions the dual field theory has *negative* energy whereas the other cases yield positive energy. The dual field theory to $AdS_7 \times S^4$ supergravity is the large N limit of the $d = 6$ $(2, 0)$ tensor multiplet theory [11] of which little is known. The field spectrum of this theory includes five scalars, a Majorana-Weyl spinor and a 2-form potential, B, with self-dual three form field strength, dB and is the intrinsic theory on N parallel M5 branes in the zero coupling limit. The calculation here seems to imply that the Casimir energy of such a theory on $S^5 \times R$ is negative. We convert the above expression to gauge theoretic quantities via

$$\alpha^{-1} = 2l_p(\pi N)^{1/3}, \quad (12)$$

with l_p being the Planck length. This yields a Casimir mass of

$$E_{Casimir(2,0)} = -\frac{5}{8}\pi^{10/3} l_p^4 N^{4/3}. \tag{13}$$

Examples of negative energy solutions in general relativity are known such as the analytically continued Reissner-Nordström solution (continued to both imaginary time and charge) although it is debatable how physical this construction is. Also, it has been noted that if one considers the Euclidean time extension of $D = d + 1$ dimensional Schwarzschild-anti-deSitter black holes, the energy corresponding to the dual gauge theory on $S^{d-1} \times S^1$ is given by [9]:

$$E = -\frac{\Omega_{d-1}\beta r_0^{d-1}\alpha}{16\pi}, \tag{14}$$

where β is the period of S^1 required to make the solution smooth at the (Euclidean) horizon. The negative sign arises from the supersymmetry breaking boundary conditions imposed along S^1. Since no such condition is imposed in the $D = 7$ case where the boundary topology is $S^5 \times R$ it is curious that a negative energy is produced.

2.1 Anomaly calculation

Recent interesting work regarding lower dimensional anomalies may be found in [13] and [14] as well as [15] where diffeomorphism techniques are utilized. Taking the trace of the full stress-energy tensor yields:

$$\begin{aligned} T &= \frac{1}{8\pi}\left[-(d-1)K - \alpha d(d-1) - \frac{R}{2\alpha} + \frac{1}{\alpha^3 2(d-2)^2}\left(\frac{d}{4(d-1)}R^2 - R_{ab}R^{ab}\right)\right. \\ &+ \frac{1}{\alpha^5(d-2)^3(d-4)}\left(\frac{3d+2}{4(d-1)}RR^{ab}R_{ab} - \frac{d(d+2)}{16(d-1)^2}R^3 - 2R^{ab}R^{cd}R_{acbd}\right. \\ &+ \left.\left.\frac{d-2}{2(d-1)}R^{ab}R_{;b;a} - R^{ab}R_{ab;c}^{;c} + \frac{1}{2(d-1)}RR_{;c}^{;c}\right)\right]. \end{aligned} \tag{15}$$

We now wish to extract the Weyl anomaly from (15). To do this the metric must be expanded in a power series in $1/r$,

$$\gamma_{ab} = r^2\gamma_{(0)ab} + \gamma_{(2)ab} + \frac{1}{r^2}\gamma_{(4)ab} + \ldots \,. \tag{16}$$

(The Einstein equations dictate that no even power appears in the expansion), The lowest order term in the expression (15) is then identified with the anomaly. This has been shown to correspond with the work of Henningson and Skenderis [16] for $d \leq 6$. The $d = 8$ case is very labour intensive and will be addressed in a later revision. Instead, we utilize a different method to calculate the anomaly which will also act as a check for future calculations.

For the following, we adopt the coordinate system of [16]. This amounts to making the transformation:

$$\begin{aligned} r &\to 1/\rho^{1/2} \\ g_{ab} &= \rho\gamma_{ab}. \end{aligned} \tag{17}$$

Using (17) the effective, renormalized action may be obtained from the following density

$$\mathcal{L} = \alpha d \int_\epsilon d\rho\rho^{-d/2-1}\sqrt{g} + \rho^{-d/2}\left(-2\alpha d\sqrt{g} + 4\alpha\rho\partial_\rho\sqrt{g}\right)|_{\rho=\epsilon}, \tag{18}$$

where $\epsilon > 0$ is the cutoff point for the ρ integration.

For even d a logarithmic term appears from the integral which arises from the bulk part of the gravitational action, i.e. the usual Einstein - Hilbert term with cosmological constant.

$$\mathcal{L} = \sqrt{g_{(0)}}\left[a_{(0)}\epsilon^{-d/2} + a_{(2)}\epsilon^{-d/2+1} + \ldots + a_{(d-2)}\epsilon^{-1} - a_{(d)}\ln(\epsilon)\right] + \text{finite terms}. \tag{19}$$

It is the coefficient of this logarithmic term ($a_{(d)}$) which is to be identified with the anomaly. By expanding $\sqrt{g}$ to order ρ^4 we may obtain the anomaly up to and including $d = 8$.

The Einstein equations are given by [16]

$$\begin{aligned} \rho(2g'' - 2g'g^{-1}g' + \mathrm{Tr}[g^{-1}g']g') + \frac{1}{\alpha^2}\mathrm{Ric}(g) - (d-2)g' - \mathrm{Tr}[g^{-1}g']g &= 0 \\ (g^{-1})^{ab}(g\prime_{ab;c} - g\prime_{ca;b}) &= 0 \\ \mathrm{Tr}[g^{-1}g''] - \frac{1}{2}\mathrm{Tr}[g^{-1}g'g^{-1}g'] &= 0, \end{aligned} \tag{20}$$

where primes denote ordinary differentiation with respect to ρ and Tr is the trace operator. All quantities are constructed with respect to g.

By using equations (20) we can determine the following relations between the g's:

$$\begin{aligned}
\mathrm{Tr}[g_{(0)}^{-1}g_{(4)}] &= \frac{1}{4}\mathrm{Tr}\left[(g_{(0)}^{-1}g_{(2)})^2\right] \\
\mathrm{Tr}(g_{(0)}^{-1}g_{(6)}) &= \frac{2}{3}\mathrm{Tr}\left[g_{(0)}^{-1}g_{(2)}g_{(0)}^{-1}g_{(4)}\right] - \frac{1}{6}\mathrm{Tr}\left[(g_{(0)}^{-1}g_{(2)})^3\right] \\
\mathrm{Tr}[g_{(0)}^{-1}g_{(8)}] &= \frac{1}{8}\mathrm{Tr}\left[\left(g_{(0)}^{-1}g_{(2)}\right)^4\right] + \frac{3}{4}\mathrm{Tr}\left[g_{(0)}^{-1}g_{(2)}g_{(0)}^{-1}g_{(6)}\right] \\
&- \frac{7}{12}\mathrm{Tr}\left[\left(g_{(0)}^{-1}g_{(2)}\right)^2 g_{(0)}^{-1}g_{(4)}\right] + \frac{1}{3}\mathrm{Tr}\left[\left(g_{(0)}^{-1}g_4\right)^2\right]. \qquad (21)
\end{aligned}$$

$g_{(2)ab}$ and $g_{(4)ab}$ may be found in [16] and are given here for reference.

$$\begin{aligned}
g_{(2)ab} &= \frac{1}{\alpha^2(d-2)}\left(R_{(0)ab} - \frac{1}{2(d-1)}R_{(0)}g_{(0)ab}\right) \\
g_{(4)ab} &= \frac{1}{\alpha^4(d-4)}\left(\frac{1}{4(d-2)}R_{(0)ab;c}^{;c} - \frac{1}{8(d-1)}R_{(0);b;a}\right. \\
&- \frac{1}{8(d-1)(d-2)}R_{(0);c}^{;c}g_{(0)ab} - \frac{1}{2(d-2)}R_{(0)}^{cd}R_{(0)acbd} \\
&+ \frac{d-4}{2(d-2)^2}R_{(0)a}^{c}R_{(0)cb} + \frac{1}{(d-1)(d-2)^2}R_{(0)}R_{(0)ab} \\
&+ \left.\frac{1}{4(d-2)^2}R_{(0)}^{cd}R_{(0)cd}g_{(0)ab} - \frac{3d}{16(d-1)^2(d-2)^2}R^2 g_{(0)ab},\right) \qquad (22)
\end{aligned}$$

and we calculated g_6 to be:

$$\begin{aligned}
g_{(6)ab} &= \frac{1}{3(6-d)}\left[4(g_{(2)}g_{(0)}^{-1}g_{(4)})_{ab} + 4(g_{(4)}g_{(0)}^{-1}g_{(2)})_{ab} - 2\left(g_{(2)}g_{(0)}^{-1}\right)^3_{ab}\right. \\
&- \mathrm{Tr}[g_{(0)}^{-1}g_{(2)}]g_{(4)ab} + \mathrm{Tr}\left[g_{(0)}^{-1}g_{(2)}g_{(0)}^{-1}g_{(4)}\right]g_{(0)ab} + \frac{1}{2}\mathrm{Tr}\left[\left(g_{(0)}^{-1}g_{(2)}\right)^3\right]g_{(0)ab} \\
&- \frac{1}{\alpha^2 2}\left[\left[g_{(4)b;a}^{c} + g_{(4)a;b}^{c} - g_{(4)ab}^{\;;c} - g_{(2)}^{cd}\left(g_{(2)db;a} + g_{(2)ad;b} + g_{(2)ab;d}\right)\right]_{;c}\right. \\
&- \left[\left(\mathrm{Tr}[g_{(0)}^{-1}g_{(4)}]\right)_{;b} - g_{(2)}^{cd}g_{(2)cd;b}\right]_{;a} + \frac{1}{2}\left[\left[\mathrm{Tr}[g_{(0)}^{-1}g_{(2)}]\left(g_{(2)b;a}^{c} + g_{(2)a;b}^{c} + g_{(2)ab}^{;c}\right)\right]_{;c}\right. \\
&- \left.\left.g_{(2)d;a}^{c}g_{(2)c;b}^{d} - g_{(2)ca;d}g_{(2)b}^{d\;;c} + 2g_{(2)ca}^{\;;d}g_{(2)b;d}^{c}\right]\right]. \qquad (23)
\end{aligned}$$

We now use the expansion of g to order ρ^4 to calculate the anomaly for $d=8$. To this order:

$$\sqrt{g} = \sqrt{g_{(0)}}\left[1+\frac{1}{2}\mathrm{Tr}A-\frac{1}{4}\mathrm{Tr}A^2+\frac{1}{6}\mathrm{Tr}A^3-\frac{1}{8}\mathrm{Tr}A^4+\frac{1}{8}(\mathrm{Tr}A)^2\right.$$
$$-\frac{1}{8}\mathrm{Tr}A\mathrm{Tr}A^2+\frac{1}{12}\mathrm{Tr}A\mathrm{Tr}A^3+\frac{1}{32}(\mathrm{Tr}A^2)^2$$
$$\left.+\frac{1}{48}(\mathrm{Tr}A)^3-\frac{1}{32}(\mathrm{Tr}A)^2\mathrm{Tr}A^2+\frac{1}{384}(\mathrm{Tr}A)^4\right], \qquad (24)$$

where

$$A \equiv \rho g_{(0)}^{-1}g_{(2)}+\rho^2 g_{(0)}^{-1}g_{(4)}+\rho^3 g_{(0)}^{-1}g_{(6)}+\rho^4 g_{(0)}^{-1}g_{(8)}. \qquad (25)$$

The anomaly is now given by studying terms of $\mathcal{O}\left(\rho^4\right)$ and using (21):

$$\begin{aligned} a_{(8)} \quad = \quad & -\frac{1}{16}\mathrm{Tr}[(g_{(0)}^{-1}g_{(2)})^4]-\frac{1}{8}\mathrm{Tr}[g_{(0)}^{-1}g_{(2)}g_{(0)}^{-1}g_{(6)}]+\frac{5}{24}\mathrm{Tr}[(g_{(0)}^{-1}g_{(2)})^2g_{(0)}^{-1}g_{(4)}] \\ - \quad & \frac{1}{12}\mathrm{Tr}[(g_{(0)}^{-1}g_{(4)})^2]-\frac{1}{12}\mathrm{Tr}[g_{(0)}^{-1}g_{(2)}]\mathrm{Tr}[g_{(0)}^{-1}g_{(2)}g_{(0)}^{-1}g_{(4)}] \\ + \quad & \frac{1}{24}\mathrm{Tr}[g_{(0)}^{-1}g_{(2)}]\mathrm{Tr}[(g_{(0)}^{-1}g_{(2)})^3]+\frac{1}{128}(\mathrm{Tr}[(g_{(0)}^{-1}g_{(2)})^2])^2 \\ - \quad & \frac{1}{64}(\mathrm{Tr}[g_{(0)}^{-1}g_{(2)}])^2\mathrm{Tr}[(g_{(0)}^{-1}g_{(2)})^2]+\frac{1}{384}(\mathrm{Tr}[g_{(0)}^{-1}g_{(2)}])^4. \end{aligned} \qquad (26)$$

In general, up to constant coefficients, the anomaly to d dimension is given by all combinations of TrA up to $A^{d/2}$.

3 Conclusion

We considered here a counterterm subtraction technique to study stress-energy tensors in higher dimensional gravity. We find that the counterterm method consistently produces correct black hole masses for $d \leq 8$ and therefore this is a most useful technique even when there is no reference background with which to renormalize the energy. The calculations also produce non-zero vacuum energies in the $r_0 = 0$ limit. These are interpreted as Casimir energies of the boundary field theory [4] which includes a *negative* energy contribution. We also find, by identifying logarithmic divergences in the gravitational action, the Weyl anomaly in the $d = 8$ case. It will be interesting to see if the counterterm technique produces a similar expression for the anomaly.

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